



ON THE DEVELOPMENT OF A NUMERICAL CALCULATION OF A ROTATION THE WIND TURBINE DARRIEUS

RUSTEM MANATBAYEV, ASSYLBEK TULEPBERGENOV, SALTANAT BERGALIEVA,
KALASOV NURDAULET

Abstract: *In this paper the mathematical model of vertical-axis Darrieus wind turbine (carousel type etc.) is described. The paper presents a mathematical formulation of the problem, the angular velocity calculating method of Darrieus wind power generating apparatus when exposed to the incoming flow and the obtained results from the calculation. According to the calculations, below the established methodology, the reliable results that well represents the physics of the phenomenon. The mathematical model and its numerical implementation and the results may be useful to further improve the mathematical description of the problem in the design and construction of vertical-axis wind turbines.*

Key words: *wind turbine, angular velocity, calculation, iteration, mathematical models, moment of inertia, the lift, drag force analysis*

1. Introduction

Propeller wind turbines with a horizontal axis of rotation are the most known and widely used. Along with them various types of devices with a vertical axis of rotation have been proposed.

The main advantages of the two vertical-axis wind turbines (wind turbine called "Darrieus", invented by French engineer Darrieus in 1925) compared to horizontal-axis are:

a) the wind direction does not matter (so there is no need for a mechanism for the orientation of the wind rose is facilitated construction and reduced gyroscopic loads simplified transmission system for electricity, which can be located at the ground level);

b) power generator, and other major components of the unit are located on the ground level, which reduces the requirements for the tower, and easy maintenance and repair. Thus, there is free access to the electrical parts for maintenance and repair works [1-3].

2. Description of the force computation methods

The rotor has a relatively small initially, but most of rapidity, because of this - a relatively high

power density, referred to its weight or value. Darrieus wind turbine is operated by the lifting force of the working vane as aircraft wings symmetrical with respect to the type of the chord profiles. The blades are at a distance r_0 from the rotation shaft and connected there to in one of two ways: with or swings "troposkino". Furs are called flat wings span, the ends of which are attached workers wings (turbine blades) with the letter "T" or "I" so that the chord of the blades was tangential to a circle of radius r_0 (Fig.1a). Method of attachment "troposkino" is that the elastic blade is folded flat to form a bow and both ends attached to the top and bottom of the rotation shaft. When the turbine blades are forced to take the form of a rotatable slack rope - troposkino (Fig.1b).

Vertical-axis wind generators can be classified by their aerodynamic and mechanical characteristics. By definition, all vertical axis machines have a common characteristic, which consists in that the aerodynamic bearing elements forming rotors move around a vertical axis, and their points describe trajectories lying generally horizontal planes [4-5].

Consider the interaction of wind turbines with fixed air flow [3-5]. Fig.2 schematically shows

the four most important positions of the working blades rotating at a constant angular velocity ω .

The angle $\theta \in [0, 2\pi]$ is measured from the x coordinate as the zero position of swing. Thus, the angle θ determines the position of the working of the blade and the forces acting on it along the circle described by the workers during the rotation of turbine blades.

As seen in Fig.2 at points A and C, the angle of attack (α) is zero because of the parallelism of the wind velocity vector \vec{V} and the vector of the linear velocity \vec{U} of rotation of the turbine.

The velocity vector of attack can be written as the sum of the vector wind speed \vec{V} and inductive linear velocity vector \vec{U} of the working of the blade with the opposite sign

$$\vec{W} = V \sin \theta \vec{e}_1 + (\omega r_0 + V \cos \theta) \vec{e}_e \quad (1)$$

where, ω - angular speed of rotation of the turbine, r - the length of the swing H - rotor, V - inductive wind speed. Below it will be shown how to determine its value.

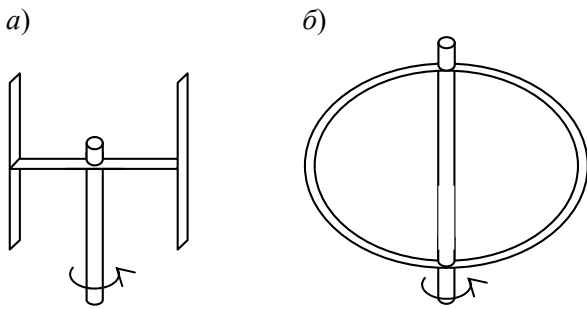


Fig. 1. Schematic of the wind turbine carousel

The angle of attack is expressed by the following formula

$$\operatorname{tg} \alpha = \frac{(\vec{W}, \vec{e}_1)}{(\vec{W}, \vec{e}_e)} = \frac{V \sin \theta}{V \cos \theta + r_0 \omega}$$

or

$$\alpha = \operatorname{arctg} \left(\frac{V \sin \theta}{V \cos \theta + r_0 \omega} \right) \quad (2)$$

Introducing the parameter rapidity

$Z = \frac{r_0 \omega}{V}$, we can get

$$\alpha = \operatorname{arctg} \left(\frac{\sin \theta}{\cos \theta + Z} \right) \quad (3)$$

a) lift force of the working blade profile

$$\vec{R}_L = C_L(\alpha) \rho \frac{W^2}{2} h dz \vec{e}_L, \quad (4)$$

where in $C_L(\alpha)$ - lift coefficient, h - height of the section chord, dz - blade adjustment element, \vec{e}_L - the unit vector in the direction of lift of the wing,

b) the resistance force

$$\vec{R}_D = C_D(\alpha) \rho \frac{W^2}{2} h dz \vec{e}_w, \quad (5)$$

Coefficients $C_D(\alpha)$, $C_L(\alpha)$ can be life represented by the formulas related to the angle of attack and are determined experimentally.

Let's write down the elementary moment of aerodynamic forces operating on an element of the dz blade at change $\theta \in [0, 2\pi]$ it is equal

$$dM = r_0 \left[(\vec{R}_D, \vec{e}_\theta) + (\vec{R}_L, \vec{e}_\theta) \right] = \rho \frac{W^2}{2} h dz r_0 \left[C_D \cos(\alpha) + C_L \sin(\alpha) \right] \quad (6)$$

where z - the third axis of orthogonal Cartesian system of coordinates. Working blades of the N-rotor

are located parallel to the third axis z .

Value W^2 we will receive, using a formula

$$W^2 = (\vec{W})^2 = V^2 \sin^2 \theta + (\omega r_0 + V \cos \theta)^2 \quad (7)$$

Let's divide cover blade into surface element dx , respectively, angle $d\theta$ subtended at this surface element, obviously, will be equal to

$$dx = r_0 \sin \theta d\theta; \quad d\theta = -\frac{dx}{r_0 \sin \theta},$$

where $d\theta$ - elementary angle of rotation around the axis Oz , covered by blade while moving from x to $(x+dx)$

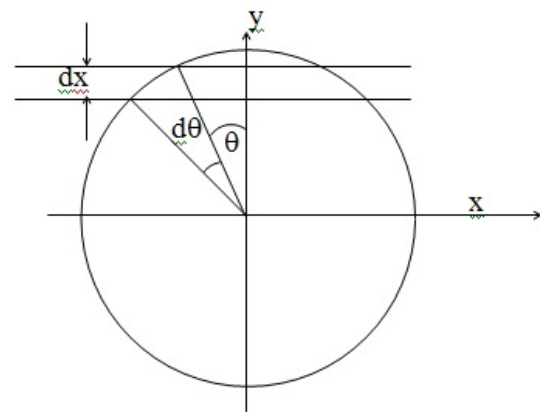


Fig. 2. Schematic moving view of current tube through surface element swept by turbine surface

Time share, spent for one rotation while moving of the element $dx dz$ is equal to

$$\begin{aligned} rd\theta &= Vd\tau = \frac{2\pi}{T} r_0 d\tau, \\ d\theta &= \frac{2\pi}{T}, \quad \frac{d\theta}{T} = \frac{d\theta}{2\pi}. \end{aligned} \quad (8)$$

In formula (8): T – period, $d\tau$ – blade element time spent in elementary tube with side surface $dx dz$. Thus, $R'(\theta)$ – an instantaneous value of propulsive force, which operates on the element in the present position of blade at its propulsion velocity, then averaged value of force $\bar{R}_x(\theta)$, operating on the element is equal to

$$\begin{aligned} \bar{R}_x(\theta) &= R'(\theta) dx \quad \text{here} \\ dx &= r_0 \sin \theta d\theta; \end{aligned} \quad (8')$$

where is determined by a formula (9'), or in more detail

$$R_x = \frac{1}{2\pi} \int_0^{2\pi} \bar{R}_x d\theta \quad (9)$$

$$\begin{aligned} R_x &= \frac{1}{2\pi} \int_0^{2\pi} \bar{R}_x d\theta = \\ &= \frac{1}{2\pi} \int_0^{2\pi} \rho \frac{W^2}{2} \frac{hB(\theta)}{r_0 \sin(\theta)} d\theta = \\ &= \frac{\rho h}{4\pi r_0} \int_0^{2\pi} \frac{W^2 B(\theta)}{\sin(\theta)} d\theta \end{aligned} \quad (10)$$

here expression (8) it is used for R_x and

$$B(\theta) = C_L \sin(\theta - \alpha) - C_D \cos(\theta - \alpha)$$

Having used determination of coefficient of C_p , are fair for $0 < \theta < \pi$, it is possible to write down

$$\begin{aligned} C_p &= \frac{R_x}{\frac{1}{2} \rho V^2 S} = \\ &= \frac{h}{2\pi r_0} \int_0^{2\pi} \left(\frac{W}{V} \right)^2 \frac{B(\theta)}{\sin(\theta)} d\theta \end{aligned} \quad (11)$$

Let's choose in a wind stream a tube of the current interacting from AVSD (fig.3) moving on a circle of the working blade. Formula for inductive speed through aerodynamic characteristics wind turbine

$$\frac{V}{V_\infty} = \frac{1}{1 + \frac{h}{8\pi r_0} \int_0^{2\pi} \left(\frac{W}{V} \right)^2 \frac{B(\theta)}{\sin(\theta)} d\theta} \quad (12)$$

The formula (12) in combination with (1) and (2) gives iterative algorithm for determination of inductive speed by V at windward side of a rotor to Darrieus, in the case under consideration, when the turbine as a whole is clasped by one tube.

Formula (12) is valid in downwind side of the rotor when $\theta \in [\pi, 2\pi]$, but instead V_∞ it is necessary to take $V_1 = 2V - V_\infty$.

In downwind side let's replace $\theta \in [\pi, 2\pi]$ blade movement all values in formulas (1), (2) and (12) and write down with superscript «'».

Thus, we obtain the following formula for downwind side of the rotor,

$$W' = \sqrt{(V' \sin \theta)^2 + (r_0 \omega + V' \cos \theta)^2}; \quad (13)$$

$$\alpha' = \arctg \left(\frac{V' \sin \theta}{V' \cos \theta + r_0 \omega} \right), \quad (14)$$

$$\frac{V'}{2V - V_\infty} = \frac{V'}{V_1} = \frac{1}{1 + \frac{h}{8\pi r_0} \left(\frac{W'}{V'} \right)^2 \frac{B'(\theta)}{|\sin \theta|}}, \quad (15)$$

$$B'(\theta) = C_L(\alpha') \sin(\theta - \alpha') - C_D(\alpha') \cos(\theta - \alpha').$$

The iteration method is also applied when determining an interference velocity on downwind side.

When carrying out calculations by the method considered above the iterative program in the Fortran language was used and realized on the computer. For carrying out calculations by us on the basis of aerodynamic characteristics of the alary NASA-0021 processing was carried out and empirical formulas of dependence of coefficients of carrying power (C_L) and force of resistance (C_D) from an angle of attack are received,

$$C_L = -0,00011 \cdot \alpha^3 + 0,0023 \cdot \alpha^2 + 0,0633 \cdot \alpha \quad (16)$$

$$C_D = 0,0005 \cdot \alpha^2 - 0,002 \cdot \alpha + 0,0129 \quad (17)$$

Let's write down expressions for the rotating moment of the turbine for windward and lee side propeller at $\theta \in [0, 2\pi]$

Finally, the velocity field at H-rotor turbine is identified theoretically. The results of these calculations are given in figures 4-8 (for $Z=2; 3; 5; 6; 7$). Conspicuous is the fact that minimum value of velocities shift left from the y-axis. Since we have chosen clockwise rotation, then such a velocity distribution formula is justified. Counter airstream in quadrants VOC and AOD creates a greater resistance, and in quadrants AOV and COD airstream direction is the same as the direction of blade movement and has less resistance (Fig. 3).

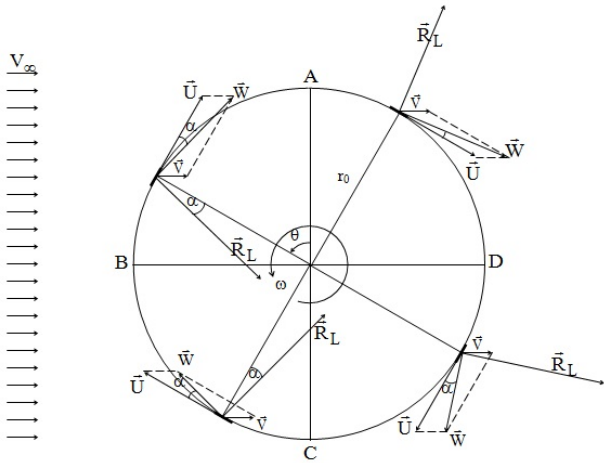


Fig. 3. Rotating counterclockwise by a working wind turbine blades

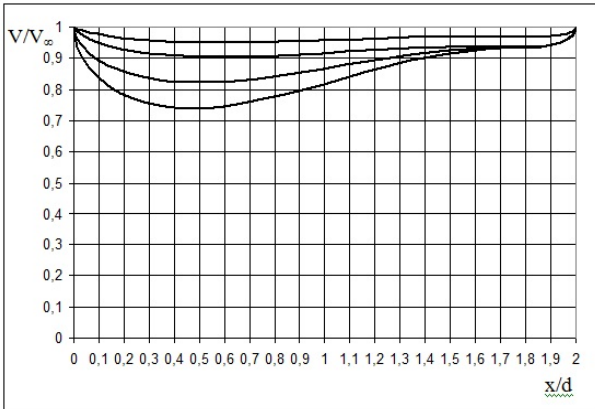


Fig. 4. Distribution of air-stream velocity while passing through the trubine (Z=2)

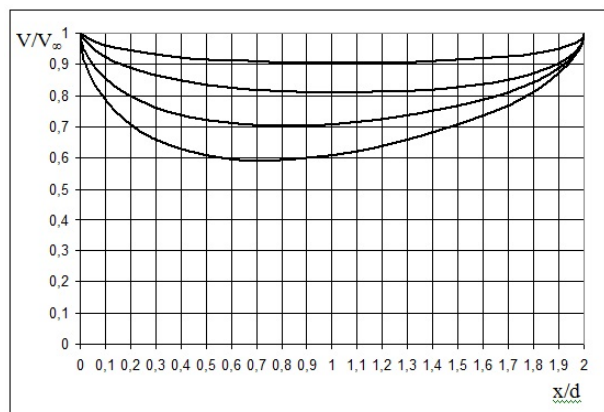


Fig. 5. Distribution of air-stream velocity while passing through the trubine (Z=3)

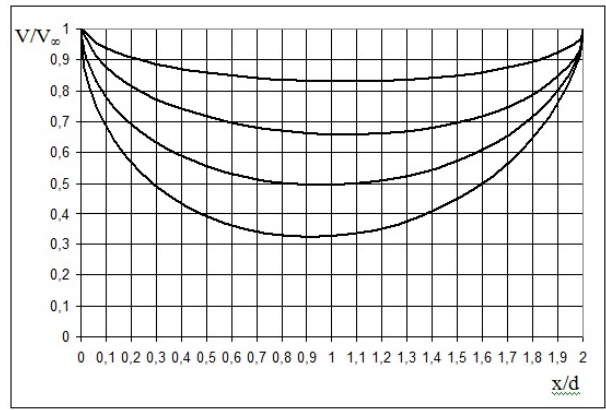


Fig. 6. Distribution of air-stream velocity while passing through the trubine (Z=5)

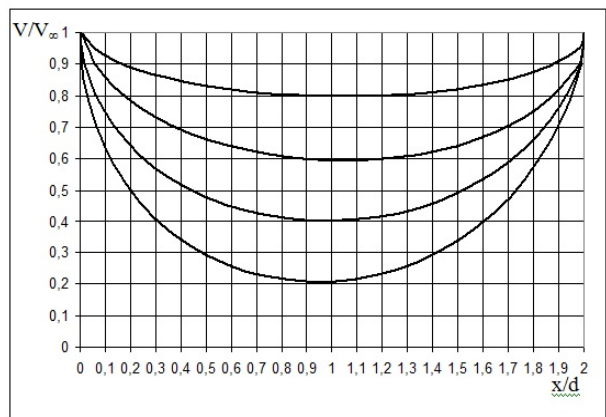


Fig. 7. Distribution of air-stream velocity while passing through the trubine (Z=6)

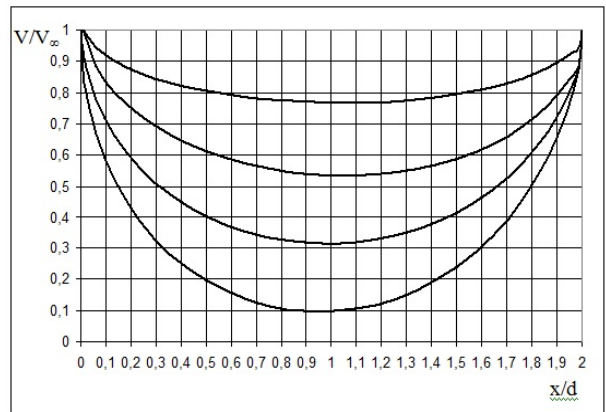


Fig. 8. Distribution of air-stream velocity while passing through the trubine (Z=7)

Let's consider windward side of wind driven propeller when $\theta \in [0, \pi]$.

We write the expression for rotary moment of turbine for this case

$$M_1 = \frac{nr_o H}{\pi} \times \int_0^\pi \rho \frac{W^2}{2} (C_L(\alpha) \sin \alpha - C_D(\alpha) \cos \alpha) d\theta \quad (18)$$

where, n – number of rotor blade, H – length of cover blade.

If to take $n=1$, i.e. take into account an interaction of one of cover blades with airstream, then it is possible to perform various characteristics computation of work turbine.

Examples of such computation are given in figures 28-42. In case n different from one the above obtained results should be multiplied by constant number n . The power has chosen from the turbine wind when passing of cover blade on semicircle from A to C (windward side of wind turbine)

$$P_1 = M_1 \omega.$$

It is also similar in downwind side when $\theta \in [\pi, 2\pi]$ is changed, formula for rotary moment of turbine is written as following

$$M_2 = \frac{Nhr_0H}{\pi} \times \int_{\pi}^{2\pi} \rho \frac{W'^2}{2} (C_L(\alpha') \sin \alpha' - C_D(\alpha') \cos \alpha') d\theta \quad (19)$$

where W' – incidence rate on cover blade in downwind side of wind turbine.

Wind turbine power in downwind side

$$P_2 = M_2 \omega.$$

The wind flows to windward side of rotor with power,

$$N_1 = \rho \frac{V_{\infty}^3}{2} 2r_0H.$$

And in downwind side, loss of power is considered when passing the windward side of rotor, thus

$$N_2 = \frac{1}{\pi} \int_0^{\pi} \rho \frac{V_2^3}{2} Hr_0 \sin \theta d\theta,$$

Therefore coefficients of wind energy use of windward and downwind side of wind turbine Darrius are determined by following formula

$$\xi_1 = \frac{P_1}{N_1}, \quad \xi_2 = \frac{P_2}{N_2},$$

Values of M_1 and M_2 moments, P_1 and P_2 turbine power, also power of N_1 and N_2 blade wind stream are determined using the iteration method on computer with 1° step.

In this work in order to determine the dependency of aerodynamic parameters of "Darrius" wind turbine on θ angle computation was performed using multitubular model of tube

theory and the program on Fortran language was drawn up.

This has made it possible to find coefficients values of wind energy use in windward side (ξ_1), as well as in downwind side (ξ_2) depending on turbine specific speed (Z).

If to take the dependency of general wind energy derivation coefficient equal to 0,45 [29], then $\xi_1=0,26$, and $\xi_2=0,19$.

At the maximum value of wind energy coefficient ξ_{\max} the wind energy of 26% is derived from windward side. The same amount of energy in percentage wise is derived from downwind side of turbine. However numerical values of ξ_1 and ξ_2 will differ, since when wind flows to downwind side of turbine the wind power is reduced due to transfer of some of its energy to windward side of turbine.

In the known work [4] the maximum value of wind energy derivation coefficient by H-rotor is slightly below. But also in this case value equality of ξ_1 and ξ_2 19% percentage wise still stands. The numerical values, accordingly, $\xi_1=0,19$, $\xi_2=0,16$.

Let's write down the formula for rotation moment

$$M = \frac{Nhr_0H}{\pi} \int_0^{2\pi} \rho \frac{W^2}{2} (C_L \sin \alpha - C_D \cos \alpha) d\theta. \quad (20)$$

In formula (27) N – number of rotor blade, H – rotor height.

The power has chosen from the wind on the windward side of the rotor is equal to

$$P = M \omega.$$

The wind flows to windward side of rotor with energy

$$N = \rho \frac{V_{\infty}^3}{2} 2r_0H.$$

All these values are calculated on ECM and we obtain each member coefficient

$$\xi = \frac{M \omega}{N}.$$

For determination of angular speed of rotation of a rotor to Darrius, at influence of a wind stream we apply the theorem of change of the kinetic moment of mechanical system [4,5] and looks like

$$\frac{dL_z}{dt} = M_{\text{turb}} + \sum M_i \quad (21)$$

where L_z – the kinetic moment of the wind turbine concerning an axis z . M_{turb} – the rotary moment created by working blades of the turbine, M_i – the moment of various forces of resistance.

For the turbine to Darrieus with two direct blades it is had

$$I = \frac{2}{3} r_0^2 m_{str} + r_0^2 m_b + r_{sh}^2 m_{sh} \quad (22)$$

where r_0 – the distance from the rotational axis to the blades, r_{sh} – radius of the shaft, m_{str} , m_b , m_{sh} – the masses of strides, blades, shaft rotation.

The time difference will express

$$\omega = \frac{2\pi}{T} = \frac{d\theta}{dt} \quad (23)$$

where $d\theta$ - corresponds to the angle of rotation of the working of the blade to the z axis in the time interval dt ;

3. Results

In (21), substituting (23) and writing in difference form, we get

$$\omega^{n+1} = \omega^n + \frac{(R_L \sin \alpha - R_D \cos \alpha) r_o + \sum M_i}{I \omega^n} \times (\theta^{n+1} - \theta^n) \quad (24)$$

and where ω^{n+1} and ω^n - respectively, the angular velocity of the turbine at the time t^{n+1} and t^n . Thus, the determination of the angular velocity ω will continue until it converges to its single value (see Fig. 9-13). As seen in Fig.13, without resistance, based can not be ignored, the turbine makes enough vibrational motion, and given the resistance and bring it to its limit, the oscillation almost vanishes (see Fig.4). The results of this work and the development of his technique will be useful for the design work for the establishment of industrial designs windmill carousel.

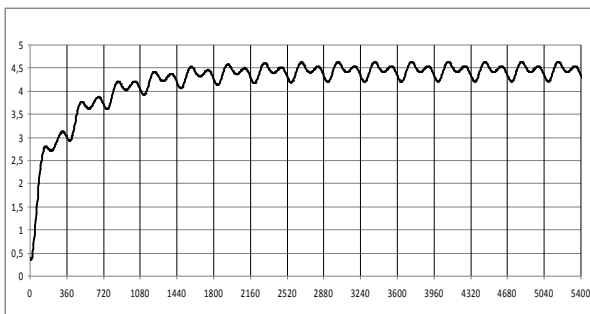


Fig. 9. Graph of the angular velocity ω on the position θ of the moving blades working at a relatively low value of $I = 0,5$ and without resistance to the turbine

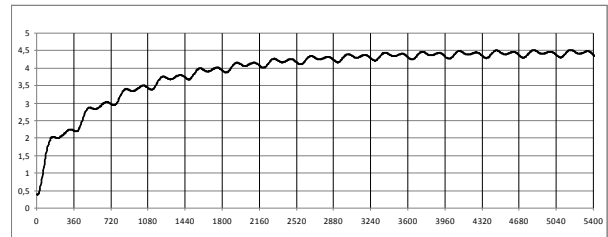


Fig. 10. Graph of the angular velocity dependency on the position of moving cover blade at relatively small value $I=1,5$ and without considering resistance of the turbine

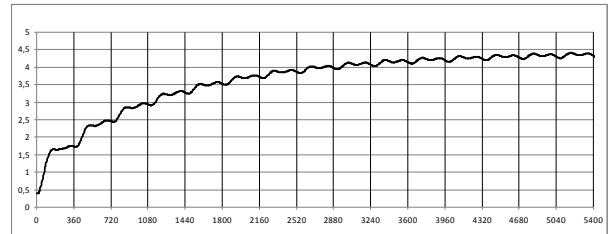


Fig. 11. Graph of the angular velocity dependency on the position of moving cover blade at relatively small value $I=2,5$ and without considering resistance of the turbine

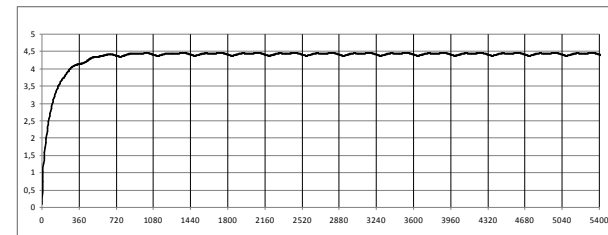


Fig. 12. Graph of the angular velocity dependency on the position of moving cover blade at $I=0,5$ and taking into account the resistance of the turbine 2,5%

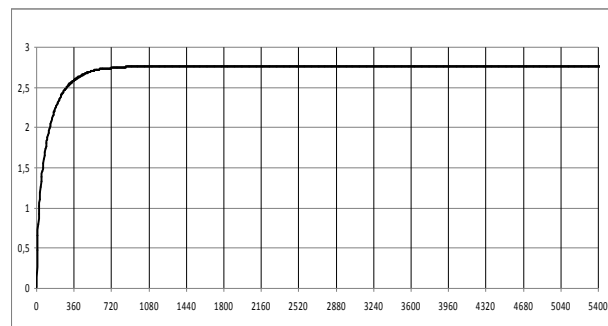


Fig. 13. Graph of the angular velocity ω on the position θ of the moving blade at work $I = 0,5$, and taking into account the resistance of the turbine 10%

4. Conclusions

The mathematical model of the angular velocity determination of the carousel type wind turbine and the calculation method and its numerical implementation are developed in this paper.

The angular velocity change, i.e. the rotation of the turbine is due to the action of the air flow, moreover it depends on the size and weight (moment of inertia) of the turbine itself.

The developed method of calculation is implemented on a computer in the programming language C ++. The series of numerical experiments were conducted by the developed program and graphs were drawn on the basis of the obtained numerical calculations. The frequency of the angular velocity change is shown on the obtained by numerical calculation graphs (see Fig. 3,4), which gives good agreement with the physics of the phenomenon of the problem.

The obtained results of numerical calculation are useful for the carousel type wind turbine design and further deep theoretical study, for example, non-stationary mode of the wind turbine rotation.

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Department of Thermal Physics and
 Technical Physics
 Physico-Technical Faculty
 Al-Farabi Kazakh National University,
 71 Al-Farabi, 050000 Almaty
 KAZAKHSTAN
 E-mail: rustem1977@mail.ru
 E-mail: asylbek12@mail.ru